

Minnesota State High School Mathematics League

Individual Event

2006-07 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

No Calculators in this Event

1. Using s and t to represent the present ages of Sally and Tom, write an equation that expresses the statement that three years ago, Sally was twice the age that Tom will be next year.

2. Solve the system below for x , y , and z . The fastest way to do this is to use the y -term in the second equation to eliminate the y -term in the third equation, and then proceed by back substitution.

$x =$
 $y =$
 $z =$

$$x - 6y + \frac{6}{7}z = -9$$

$$y - \frac{2}{3}z = 2$$

$$3y - 4z = -1$$

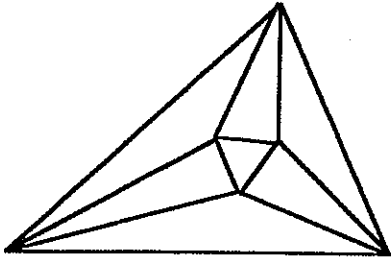
3. The equation

$$\begin{vmatrix} x & y & 1 \\ 4 & 1 & 1 \\ -1 & 7 & 1 \end{vmatrix} = 0$$

has a graph that is a straight line. What is the slope of the line?

4. A company makes school sweatshirts and sweatpants. Five sweatshirts and six sweatpants cost a total of \$147. For orders totaling more than 30 items, the company reduces the total price of the sweatshirts by 40% and cuts the total price of the sweatpants in half. Forty sweatshirts and forty sweatpants, therefore, cost a total of \$578. Find the original cost of a single pair of sweatpants. [Massachusetts Math League, March 2006].

Name _____ Team _____



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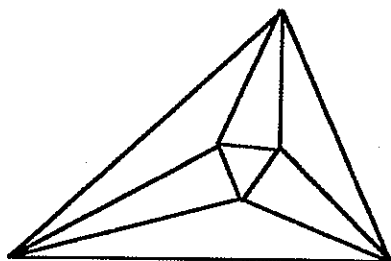
Individual Event

2006-07 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. A rhombus with sides of length 4 has a long diagonal of length 6. What is the length of the short diagonal?
- _____ 2. A rhombus with sides of length 4 has acute angles of 60° . What is the length of the long diagonal?
- _____ 3. A rhombus with sides of length 4 has acute angles of 45° . What is the length of the long diagonal?
- _____ 4. In 2005, as part of the paper she wrote for the Minnesota Scholars of Distinction program, Jean Huang proved that if on each side of a rhombus we construct a square that is exterior to the rhombus, then the centers of the four squares will be vertices of a new square. Starting with the rhombus described in Problem 2, what will be the length of the sides of the new square?

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2006-07 Event 3C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. In $\triangle ABC$ (Figure 1), $\angle BAC = 30^\circ$, $AB = 1$, and $BC = 2$. Find the measure of $\angle ACB$, accurate to one tenth of a degree.
- _____ 2. Refer again to $\triangle ABC$ in figure 1 where $AB = 1$ and $BC = 2$. Suppose that $\angle BAC \neq 30^\circ$, that instead $\angle ABC = 120^\circ$. Find AC .
- _____ 3. If $\alpha = \tan^{-1} \frac{m}{n}$ and $\beta = \tan^{-1} \frac{n-m}{n+m}$ where m and n are positive integers, find in simplest terms $\tan(\alpha + \beta)$.
- _____ 4. Find three distinct numbers of the form $a + bi$ that are cube roots of $-i$.
- _____
- _____

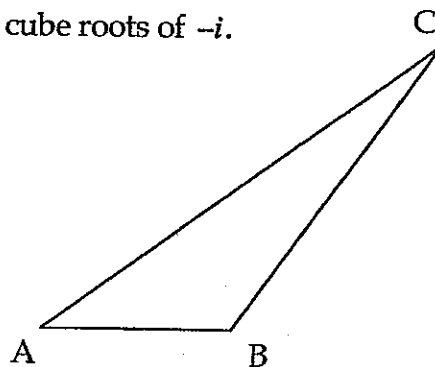
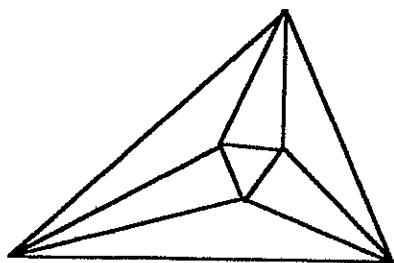


Figure 1

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Minnesota State High School Mathematics League

Individual Event

2006-07 Event 3D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. In each case you are given a graph and a list of equations, one of which corresponds to the graph. Circle what you believe to be the corresponding equation. You have 12 minutes for this event.

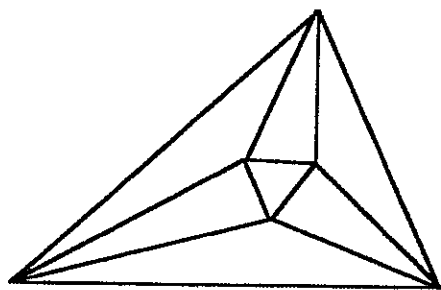
_____ 1. Find x , given $2^{-\sqrt{x}} = \frac{1}{8}$.

_____ 2. If $x = -2$, what is the value of $-x^x|x|3^{-x}$?

_____ 3. Find $\log xy$, given that $\log x^2 y^3 = 1$ and $\log x^5 y^2 = 8$

4. Consider the equation $4^x - 2^x - 6 = 0$. By choosing the correct integer k , x may be expressed in the form $x = n \log_k n$. Write x in this form, clearly indicating both k and n .

Name _____ Team _____



Minnesota State High School Mathematics League

Team Event

2006-07 Meet 3

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. The inequalities below define a region D in the xy -plane. You get 1 point, up to a maximum of 4, for each lattice point (a point with integer coordinates) that you find interior (not on the boundary) to D .

$$x + y \leq 6, \quad 4x + y \geq 8, \quad x + 6y \geq 6$$

2. Refer again to the region D defined in Problem 1. Consider the function

$$f(x, y) = 6 - 3x - \frac{1}{2}y. \text{ What is the maximum value of } f \text{ for points in } D \text{ (all of } D, \text{ not just its interior)?}$$

3. Two poles stand on level ground, one 45 feet high, the other 75 feet high. From the top of each pole, a wire runs to the base of the other pole (Figure 3). How high above the ground does the intersection of the wire occur?

4. Figure 4 shows a right $\triangle ABC$ with a hypotenuse of length h , and a second right $\triangle EBD$ with hypotenuse k . Let $\alpha = \angle BAC$ and $\beta = \angle BED$, with $\beta > \alpha$. From the point F where AC and ED intersect, a line is drawn to G on AB , making $FG \perp AB$. Express the length of FG in terms of h , k , α and β .

5. Solve the following system for x , y , and z .

$$2x + 3y - 2z = -4$$

$$-3x + 9y - 5z = 4$$

$$5x + 6y + 3z = 8$$

$$x = \quad y = \quad z =$$

6. Using logarithms with base 10, $\log MN = 1.62583$, and $\log \frac{M}{N} = 0.60206$. Find M , which is an integer.

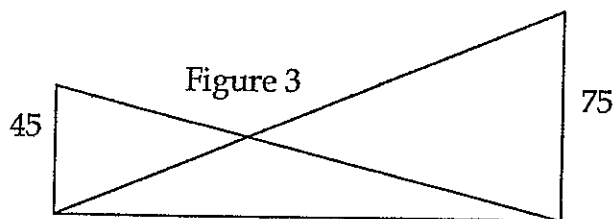
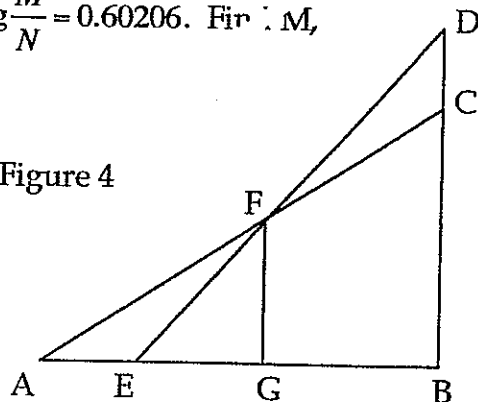


Figure 4



Team _____